Repairable system reliability: recent developments in CBM optimization

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Abstract

This paper presents some developed and implemented theory which can be applied to optimization of condition-based maintenance decisions within the context of physical asset management. We examine a replacement problem for a system subject to stochastic deterioration. In particular, the analysis of a preventive replacement policy of the control-limit type for a system subject to inspections at discrete points of time is presented. Cox's Proportional Hazards Model with Weibull baseline hazard function and time dependent stochastic covariates, reflecting the item's condition, is used to describe the hazard rate of the system. Statistical results are then blended with economic or performance considerations to establish long-run optimal maintenance strategies.
The structure of the decision-making software EXAKT™ is presented, and recently added optimization and prognostic capabilities are described.

Keywords: condition-based maintenance, proportional-hazards model, Markov process, cost and availability optimization, decision software

1. Introduction

Condition Monitoring (CM) has become a recognized tool for assessment of operational state of industrial equipment. Maintenance decisions, such as when to take an action and what type of action to take, can be made based on analysis of CM information. Examples of CM information that can be utilized include, but are not limited to: Vibration Measurement and Analysis, Infrared Thermography, Oil Analysis and Tribology, Ultrasonics, Motor Current Analysis, etc. [1]
In order to obtain useful interpretation, data collection about the operational condition of the item should be followed by proper analysis, capable of extracting meaningful and reliable information from it.
Control charts are one of the most commonly applied techniques for interpretation of CM data. At each inspection, levels of some measurements are compared with the corresponding predefined "warning limits" and judgment is made based on the outcome. The method has been applied for several decades and proved to be a helpful and simple to understand technique.
However, control charts leave several important questions unanswered. Among the variety of measurements related to the items condition that one can collect, which ones should be paid attention to? What if there is no single variable that can provide information on true condition of the equipment? What are the optimal warning limits and should these limits change with operating age of the item?
The other conventional approach to maintenance planning and decision making is the

age-based strategy. The classical age replacement strategy recommends replacing an item either at failure or when it reaches a certain age. Several modifications of the classical method have been proposed (See, for example, [2]), and this paper discusses the model which extends the age-based model with addition of analysis of CM information.

The idea of taking advantage of both the CM information and the age data for modeling useful statistical characteristics of equipment represents the next step of evolution of techniques in maintenance. In the literature, such approach to optimization of maintenance decisions is referred to as the Condition-Based Maintenance (CBM) technique, and advantage is that by taking into account both the age of the item and it's history it significantly expands the space of available maintenance strategies.

The CBM Consortium research laboratory was established in 1995 at the Department of Mechanical and Industrial Engineering in the University of Toronto. The lab has developed theory that combines age and condition monitoring data with economic and/or performance data that may include the cost of failure, the cost of planned maintenance, the corresponding down times, and produces a long-run optimal maintenance decision policy. Among current activities of the project is development of software that can assist maintenance and reliability specialists to optimize decisions in CBM environment. The current state of development of the software, called EXAKT™, is presented in section 4.

The rest of this paper is organized as follows. Section 2 introduces the theory that can be used for modeling of time to failure of the item. The approach to statistical modeling based on the Proportional Hazards Model is described. Methods of obtaining the conditional lifetime distribution and some of its useful characteristics are presented. Section 3 of this paper contains description of several decision models that have been developed by the Condition-Based Maintenance Laboratory to optimize maintenance strategies based on criteria of cost or availability. Description of optimization software and an implemented approach to generalization of the presented theory to systems with multiple failure types is proposed in section 4. Finally, section 5 contains the conclusion.

2. Failure time model

In this paper we consider a replacement model in which an item is replaced with another one "as good as new", either at failure or at planned replacement. Item histories are assumed to be independent and identically distributed random processes. A history includes the information on the item's observed lifetime, censoring information and information on diagnostic variables collected at regular discrete times during the observation period.

In this paper diagnostic variables will be termed covariates. In practice, both the external variables (operating environment conditions) and internal (diagnostic) variables can be used as covariates for the analysis. The external covariates can affect the time to failure, and the internal variables can reflect the current state of the item.

Complete details on the statistical theory presented in this section are found in [3] and [4].

2.1 Statistical model

Let T be the time to failure of the item. The time-dependent condition-monitoring indicators are modeled by a non-homogeneous discrete Markov process $\{Z(t), t > 0\}$, where $Z(t) = (Z_1(t), Z_2(t), ..., Z_m(t))$ is an m-dimensional covariate process observed at regular inspections of the item. It is assumed that $Z(t)$ is a right continuous process, with left-hand limits, and that each covariate $Z_i(t)$ is a discrete numerical variable with finite number of values. Let $\{0,1,2,...,n\}$ be the finite state space of $Z(t)$. Then the overall system can be modeled by the joint process $(I(T > t), Z(t))$ ($I(*)$ being the indicator function) with transition probabilities

$$L_{ij}(x,t) = P(T > t, Z(t) = j \mid T > x, Z(x) = i)$$

For the analysis it is convenient to represent $L_{ij}(x,t)$ in the following form

$$L_{ij}(x,t) = P(T > t \mid T > x, Z(t) = j) \cdot P(Z(t) = j \mid T > t, Z(x) = i) \tag{1}$$

Then for a short interval of time $[x, x + \Delta x]$, values of transition probabilities can be approximated as follows:

$$L_{ij}(x, x + \Delta x) = [1 - h(x,i)\Delta x] \cdot p_{ij}(x, x + \Delta x) \tag{2}$$

where $p_{ij}(x,t) = P(Z(t) = j \mid T > t, Z(x) = i)$ is the conditional transition probability of the covariate process $Z(t)$, and $h(x,i)$ represents the hazard function. Details on modeling of $h(x,i)$ will be discussed in section 2 of this paper. Values of $p_{ij}(x, x + \Delta x)$ can be approximated from historical data using the maximum likelihood method. More details on estimation of transition probabilities can be found in [3].
For longer intervals transition probabilities can be derived from (2) using the Markov property:

$$L_{ij}(x, x + m\Delta x) = \sum_k L_{ik}(x, x + (m-1)\Delta x) \cdot L_{kj}(x + (m-1)\Delta x, x + m\Delta x)$$

2.2 Proportional hazards model with time-dependent covariates

In this paper the influence of CM indicators on the failure time is modeled using the Proportional Hazards Model (PHM). First proposed by Cox in 1973 [5], the PHM and its variants have become one of the most widely used tools in the statistical analysis of the lifetime data in biomedical sciences and reliability. For our analysis we consider a parametric PHM with baseline Weibull hazard function as a model for the hazard function. This model is also known as a Weibull parametric regression model. For this model

$$h(t, Z(t); \beta, \eta, \gamma) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{\sum_1^m \gamma_i Z_i(t)}$$

$$\beta > 0, \eta > 0, \gamma = (\gamma_1, \gamma_2, ..., \gamma_m)$$

The method of maximum likelihood can be applied for estimation of parameters β, η, γ of the model. For more details, please refer to [3].

2.3 Conditional distribution of time to failure

Within the framework of statistical models introduced in sections 1 and 2, the conditional reliability function of the item, given the current state of the covariate process can be expressed using (1) as follows:

$$R(t \mid x, i) = P(T > t \mid T > x, Z(x) = i) = \sum_j L_{ij}(x, t) \tag{3}$$

Once the conditional reliability function is calculated we can obtain the conditional density from its derivative. We can also find the conditional expectation of $T - t$, termed the remaining useful life (RUL), as

$$E(T - t \mid T > t, Z(t)) = \int_t^{\infty} R(x \mid t, Z(t))\, dx$$

In addition, the conditional probability of failure in a short period of time $[t, t + \Delta t]$ can be found as

$$P(\text{Survive during } [t, t + \Delta t] \mid t, Z(t)) = R(t \mid t, Z(t)) - R(t + \Delta t \mid t, Z(t))$$

For a maintenance engineer, predictive information based on current CM data, such as RUL and probability of failure in a certain period of time, can be a valuable tool for assessment of risks and planning appropriate maintenance actions.

3. Decision models

3.1 Economic decision model

The objective of the economic decision model is to develop a rule for preventive replacement that minimizes the average replacement cost per unit time due to preventive and failure replacements over a long time horizon. Let $C_p = C$ be the preventive replacement cost, and $C_f = C + K$ be the failure replacement cost, per one replacement. These costs are assumed fixed for all replacements. Let $T_d = \inf\{t \geq 0 : Kh(t, Z(t)) \geq d\}, d > 0$ define a "control-limit" policy, i.e. if $T_d < T$, perform the preventive replacement at time T_d, and if $T_d \geq T$, perform the failure replacement at time T. Let the probability of failure replacement be denoted by $Q(d) = P(T_d \geq T)$, and the expected time until replacement be denoted by $W(d) = E(\min\{T_d, T\})$. Then the long-run expected cost of replacements per unit time $\Phi(d)$ is

$$\Phi(d) = \frac{C + KQ(d)}{W(d)} \tag{4}$$

The value d^* that minimizes the right-hand side of expression (4) corresponds to the optimal control-limit policy $T^* = T_{d^*}$. Makis and Jardine in [6] have shown that for a non-decreasing hazard function $h(t, Z(t))$, rule T^* is the best possible replacement policy (See also [7]). It can be mentioned that for non-monotone hazard function,

control-limit approach can still be viewed as providing a "near to optimal" replacement policy (more discussion can be found in [3]).
For a non-decreasing hazard function the optimal risk threshold d^* that minimizes $\Phi(d)$ can be found using the fixed point iteration algorithm [6]. In general case, direct numerical search can be applied.
For the PHM model with Weibull baseline distribution, the optimal replacement rule $T^* = T_{d^*} = \inf\{t \ge 0 : Kh(t, Z(t)) \ge d^*\}$ can be interpreted as

$$T^* = \min\{t \ge 0 : \sum_{i=1}^{m} \gamma_i Z_i(t) \ge \delta^* - (\beta - 1)\ln t\}$$

where $\delta^* = \ln\left(\frac{\eta^\beta d^*}{\beta K}\right)$. The function $g(t) = \delta^* - (\beta - 1)\ln t$ can be considered as a "warning level" function for the condition of the item reflected by a weighted sum of current values of covariates. A plot of function $g(t)$ versus working age can be viewed as an economical decision chart which shows whether the data suggests that the item has to be replaced. An example of a decision chart with several inspections points can be found on Figure 1. Detailed case studies based on the model discussed in this section can be found in [8], [9].

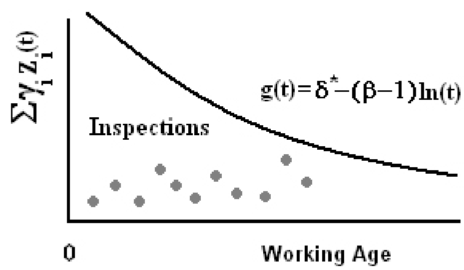

Figure 1. Sample Economical Decision Chart (for $\beta > 1$)

3.2 Availability maximization

In some cases, maintenance engineers are faced with the problem of maximization of operating availability, rather than cost. In such cases, the optimal long-run balance between downtime due to preventive replacements and downtime associated with failures can also be optimized using model (4). For the availability setting, if t_p represents the downtime required for preventive replacement, and $t_f \ge t_p$ - failure downtime, define the expected long-run availability function $\Phi(d)$ as follows:

$$\Phi(d) = \frac{W(d)}{W(d) + t_p \cdot (1 - Q(d)) + t_f \cdot Q(d)} \tag{5}$$

The value of d^* that maximizes function $\Phi(d)$ in (5) defines the optimal control-limit replacement policy $T^* = T_{d^*}$. In order to demonstrate relation of the availability model to the model described in section 1, consider the following reformulation of (5):

$$\Phi(d)=\frac{1}{1+\frac{t_p+(t_f-t_p)\cdot Q(d)}{W(d)}}$$

Clearly, the maximization problem (5) is equivalent to a minimization problem of type (4) with parameters $C=t_p$ and $K=t_f-t_p$. Methods described in section 1 can be used for calculation of the optimal hazard threshold level d^* in this model.

3.3 Balancing availability and cost

The model proposed in section 1 can also be applied for situations when consequences of downtime can be quantified and thus contribute to the overall cost of equipment maintenance. In this setting, if we define a_p and a_f to be hourly costs of downtime due to preventive and failure replacements respectively, then the model can be modified as follows:

$$\Phi(d)=\frac{(C_p+a_pt_p)\cdot(1-Q(d))+(C_f+a_ft_f)\cdot Q(d)}{W(d)+t_p\cdot(1-Q(d))+t_f\cdot Q(d)} \tag{6}$$

In the models discussed in sections 1 and 2, it can easily be seen that when $C_f\le C_p$ or $t_f\le t_p$, then the optimal strategy will always be to run to failure. For the combined model, however, some more complex rule can be derived.
In the notations introduced above, it can be shown that in order to determine whether the Run-to-Failure strategy is optimal, one can check if

$$\frac{C_f+a_ft_f}{\mu+t_f}\le\frac{C_p+a_pt_p}{\mu+t_p} \tag{7}$$

where μ is the expected time to failure under the Run-to-Failure policy. In accordance with (7), we have:

$$\frac{C_f+a_ft_f}{\mu+t_f}\le\frac{C_p+a_pt_p}{\mu+t_p}\le\frac{C_p+a_pt_p}{W(d)+t_p},\ \text{since}\ W(d)\le\mu$$

which implies

$$\Phi(d)=\frac{(C_p+a_pt_p)\cdot(1-Q(d))+(C_f+a_ft_f)\cdot Q(d)}{W(d)+t_p\cdot(1-Q(d))+t_f\cdot Q(d)}\ge\frac{C_f+a_ft_f}{\mu+t_f}=\Phi(\infty)$$

and means that the Run-to-Failure policy is the best, i.e. it provides the least long-run average cost $\Phi(\infty)$.
Condition (7) demonstrates that relations between parameters in the combined model, as far as analysis of policies is concerned, are more complex than those in previously described models. It can be seen, for example, that in some cases high cost of replacement at failure can be less important for the optimization process than the cost of downtime related to preventive replacement, even if the cost of preventive replacement is relatively low.

4. EXAKT™ - a tool for evidence-based decision making

Practical application of the theory presented in this paper requires a significant effort in collecting, preparing, filtering, analyzing the data, and using the result for

optimizing maintenance strategies. This creates a need in a software tool that will be capable of performing the above named tasks and yet remain simple to understand and operate.
The theory described in this paper has been implemented by the CBM lab at the University of Toronto to produce the software with the capabilities required for modeling and CM-based decision making.
The current state of development of the software, named EXAKT™, allows the user to:

- Create a convenient database by extracting the event and condition (inspection) data from external databases;
- Detect logical errors in the databases;
- Perform data analysis and preprocessing, using graphical and statistical analysis;
- Estimate parameters of the PHM and Markov process model. The model can be evaluated based on such statistical tests as Wald test, Log-likelihood test, Kolmogorov-Smirnov test, χ^2 test for independence of covariates and for homogeneity of the Markov process;
- Calculate and graphically present the conditional probability distribution for a given item and provide such characteristics as RUL and probability of failure in a short time period;
- Compute and save the optimal replacement policy. Alternate policies are also available based on Age and Block replacement strategies;
- Perform separate analysis for different failure modes or components of the system and create an integrated decision module;
- Make and save decisions for current records whenever it is required, using the developed decision model.

Figure 2 shows the diagram illustrating the principle of work of the software and the way it can be used in decision-making. As outlined above, the program utilizes the age data and the condition-monitoring data in order to produce a statistical model, which in turn can be used to derive useful justified predictions and/or to optimize economic considerations. It is our belief that when supplied with the results of these analyses, an engineer can make better maintenance decisions.

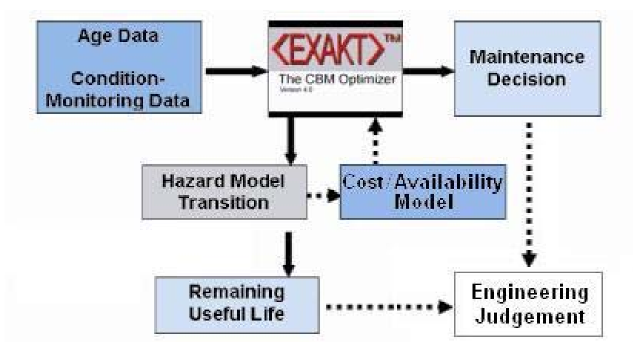

Figure 2. Principle of EXAKT™

The software also provides procedures for the checking, correction and transformation of data. A simplified programming language has been developed and included in EXAKT™ to help the user analyze the data using graphical methods and a number of statistical operations. An overview of features implemented in EXAKT™ can be found in [3] and [10].

4.1 Marginal Analysis in EXAKT™

For a multi-component system, or a system with multiple failure modes, the software has an option called Marginal Analysis. Under this option, for a single set of data, separate models can be built for different components (or failure modes) and then integrated to produce one general decision model.

Separate analyses of different components (or failure modes) can help for better planning and scheduling of preventive maintenance activities, more targeted work orders, possibilities for opportunistic preventive maintenance, etc. However, marginal analysis requires additional information on lifetime history of equipment, such as classification of events of failure, which might not always be accessible.

One of the case studies undertaken by the CBM lab was intended to analyze performance of Diesel Engines employed on ships. As many as ten different failure modes have been defined, five of which have been found related to the available condition monitoring data (oil analysis data) collected by the user over the years. If ignored, interactions between different causes of failure could have led to a conclusion that time was not a significant risk factor for the engine. At the same time, when separated, analyses of different failure modes showed that at the component level it was possible to build time-dependent statistical models and, thus, derive more targeted policies for component replacements. In terms of the system, it translated into a component replacement strategy which yielded 20%-50% of improvement (depending on the ratio of costs of planned and failure replacements) in the long-run cost per unit time as compared with the Run-to-Failure strategy.

Challenge remains to develop theory revealing relations between different components (or failure modes) within a system. This problem, among others, is one of the current research interests of the CBM lab. An approach to analysis and modeling of complex systems as well as review of literature can be found for example in [11].

5. Conclusion

The growing competitiveness in the industrial world is driving the interest in improvement of asset effectiveness. Application of condition monitoring techniques is growing and produces a challenge for researchers to develop appropriate decision making strategies. Statistical modeling of acquired data and economic considerations of maintenance activities have proven to be useful for making evidence-based decisions and building justified predictions for the future behavior of the equipment. Development of theoretical optimization models should be followed by the development of software for analysis of condition-monitoring and equipment lifetime data in order to ensure successful implementation of new techniques in the industry.

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